

Elementary Mathematics I (MTH101)

Exam Summary

- A set is any well-defined list, collection, or class of objects.
- Given a set A with elements 1,3,5,7 the tabular form of representing this set is $A = \{1, 3, 5, 7\}$.
- The set-builder form of the same set is $A = \{x | x = 2n + 1, 0 \leq n \leq 3\}$.
- Given the set $N = \{2,4,6,8,\dots\}$ then N is said to be infinite since the counting process of its elements will never come to an end, otherwise, it is finite.
- Two sets of A and B are said to be equal if they both have the same elements, written $A = B$
- The null set, $\emptyset$, contains no elements and is a subset of every set.
- The set A is a subset of another set B, written $A \subset B$, if every element of A is also an element of B, i.e. for every $x \in A$ then $x \in B$
- If $B \subset A$ and $B \neq A$, then B is a proper subset of A
- Two sets A and B are comparable if $A \subset B$ and $B \subset A$
- The power set 2^S of any set S is the family of all the subsets of S
- Two sets A and B are said to be disjoint if they do not have any element in common, i.e their intersection is a null set.
- The Union of sets A and B, denoted by $A \cup B$, is the set of all elements, which belong to A or to B or to both.
- The intersection of sets A and B, denoted by $A \cap B$, is the set of elements, which are common to A and B. If A and B are disjoint then their intersection is the Null set $\emptyset$
- The difference of sets A and B, denoted by $A - B$, is the set of elements that belong to A but which do not belong to B.
- The complement of a set A, denoted by A' , is the set of elements, which do not belong to A, that is, the difference of the universal set U and A.
- The set of real numbers and their properties is called the **real number system**.
- We denote the integers by Z; hence we can write $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$
- The integers are also referred to as the “whole” numbers.
- **Rational numbers** are those real numbers, which can be expressed as the ratio of two integers.
- We denote the set of rational numbers by Q. Therefore, $Q = \{x = \frac{p}{q} \text{ where } p \in Z, q \in Z\}$
- The natural numbers are positive integers. We denote the set of natural numbers by N; hence $N = \{1,2,3,\dots\}$
- Irrational numbers are those real numbers that are not rational. hence Q' denote the irrational numbers. Examples of irrational numbers are $\sqrt{3}, \pi, \sqrt{2}$, etc.
- The set of complex numbers is a superset of the set of real numbers.

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- The notation $x < 5$ means that x is a real number that is less than 5; hence x lies to the left of 5 on the real line.
- The absolute value of a real number x , denoted by $|x|$ is defined by the formula: $|x| = \{x \text{ if } x > 0, -x \text{ if } x < 0\}$, that is, if x is positive or zero then $|x|$ equals x , and if x is negative then $|x|$ equals $-x$.
- The absolute value of any number is always nonnegative, i.e. $x \geq 0$ for every $x \in \mathbb{R}$.
- The intersection of two intervals is an interval, that is, $A \subset \mathbb{Z}, B \subset \mathbb{Z}$ implies $A \cap B \subset \mathbb{Z}$.
- The union of two non-disjoint intervals is an interval, that is, $A \subset \mathbb{Z}, B \subset \mathbb{Z}, A \cap B \neq \emptyset$ implies that $A \cup B \subset \mathbb{Z}$.
- The difference between two non-comparable intervals is an interval, that is, $A \subset \mathbb{Z}, B \subset \mathbb{Z}, A \not\subset B, B \not\subset A$, implies $A - B \subset \mathbb{Z}$.
- Sets of the form $A = \{x | x > 1\}$, $B = \{x | x \geq 2\}$, $C = \{x | x < 3\}$, $D = \{x | x \leq 4\}$, $E = \{x | x \in \mathbb{R}\}$ are called infinite intervals and are also denoted by $A = (1, \infty)$, $B = [2, \infty)$, $C = (-\infty, 3)$, $D = (-\infty, 4]$, $E = (-\infty, \infty)$.
- Set A is **bounded** if there exists a positive number M such that $|x| \leq M$ for all $x \in A$.
- A set is called **unbounded** if it is not bounded.
- The set of real numbers, $\mathbb{R}$, contains the set of integers, $\mathbb{Z}$, Rational numbers, $\mathbb{Q}$, Natural numbers, $\mathbb{N}$, and Irrational numbers, $\mathbb{Q}'$.
- Intervals on the real line are open, closed, open-closed, or closed-open depending on the nature of the endpoints.
- A sequence is a succession of terms spanned by a rule or formula.
- A sequence may be **finite** or **infinite**.
- A **finite sequence** is one whose first and last elements are known, while an **infinite sequence** is one whose terms are uncountable.
- The general term of a sequence (formula) can be written as; $U_n = (n+1)/(2n-1)$.
- Arithmetic Sequence (A.P) is a sequence in which each term differs by a common difference.
- The last term or general term for an Arithmetic Sequence is $U_n = a + (n-1)d$ where a is the first term, n is the number of terms and d is the common difference.
- The sum of the sequence is $2S = n/2 [2a + (n-1)d] = n/2 [a + u]$, where S , is the sum of the first n –terms of the A.P sequence.
- A **geometric sequence** is a sequence in which each successive terms of the sequence are in equal ratio.
- The formula for **geometric sequence** is $U_n = ar^{n-1}$ where a is the first term, r is the common ratio, and U is the n th term.

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- There are three ways to solve a quadratic equation.
- **Factorization:** This is a typical method to solve a quadratic equation whenever the polynomial ax^2+bx+c can be easily factored.
- **Completing the Square Method:** This is a method that can be used to solve any quadratic equation. However note that $x^2+bx+(b/2)^2=(x+b/2)^2$
- **Quadratic Formula:** The formula for solving quadratic equations is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is also the most recommended method, apart from being a solution of quadratic equations, it has a lot of applications.

- The expression inside radical b^2-4ac is called the discriminant. Using the discriminant, we can tell the following without solving the equation itself.
- *Theorem.* For $ax^2+bx+c=0$ with $a \neq 0$,
 - If $b^2-4ac > 0$, then the equation has two distinct real solutions.
 - If $b^2-4ac = 0$, then the equation has only one real solution i.e equal and repeated roots.
 - If $b^2-4ac < 0$, then the equation has two complex solutions that are conjugated with each other.
- **Two roots of the Quadratic Equations** are: α and β therefore the **sum of the roots** $\alpha + \beta = -b/a$, and the **product of the roots** $\alpha\beta = c/a$.
- **MATHEMATICAL INDUCTION** is defined thus: "If a result involving natural numbers is true for 1 and its validity for a natural number k implies its validity for $k + 1$, then the formula is true for all numbers."
- A complex variable z is of the form $a + bi$, where a and b are real numbers and i is called the imaginary number.
- Suppose a complex number is $z = a + bi$, then $\bar{z} = a - bi$ is called the conjugate of the complex number.
- The Modulus of a Complex Number is $z_1 = |z_1|$
- The distance between two points $z_1 = (a_1 + b_1i)$ and $z_2 = (a_2 + b_2i)$ in a complex plane is given by: $\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$
- **Triangle inequality** of complex numbers $|z_1 + z_2| \leq |z_1| + |z_2|$, and $|z_1 + z_2| \geq |z_1| - |z_2|$
- A circle is the locus of a curve (equidistant from a point). A circle could also be described by its centre (fixed) and its radius.

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- The **radius** is the distance between the center of the circle and the circumference.
- The **equation of the circle** is $y^2 + x^2 - 2gx - 2fy + c = 0$
- **Properties of Equation of a Circle**

The coefficient of the two variables x and y must be the same;

There must be no term in xy and;

The equation must be second degree or simply in second degree of x and y respectively.